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CONTINUOUS MODAL CONTROL OF LINEAR MULTICOUPLED OBJECTS*

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A modal control method is considered in which the spectrum of the closed-loop system is continuously deformed in such a way that the spectrum of the open-loop object transforms into the desired spectrum. The algorithm of the continuous modal control is synthesized. The conditions for spectral control in the method are obtained. The approach is based on similar ideas to those in /1/, but a different class of controls is considered here. Moreover, by using the appratus of Lyapunov functions, specified in the one-parameter family of the deformed spectrum, the deviation between the required spectrum and the closed-loop system spectrum can be minimized in the Euclidean metric, in the case when the wanted spectrum cannot be obtained in the closed-loop system.

1. Formulation of the problem. Suppose we are given the linear controlled object

$$x^{\cdot}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) \tag{1.1}$$
$$x \in R^n, \quad u \in R^m, \quad y \in R^l$$

where x is the state vector, u is the control vector, y is the vector of observed variables A, B, C are constant matrices of suitable dimensionless, and R^n is a linear n-dimensional space over the real number field. We shall in future assume that the spectrum of the object (1.1) is simple and contains no multiple poles. We define the class of controls by

$$u_\alpha(t) = \Big(\int_0^\alpha G(\xi)\, d\xi\Big) y(t), \quad G \in R^{m\times l} \tag{1.2}$$

where G is a matrix function of the scalar variable ξ, and $\alpha \geqslant 0$ is a parameter. The dynamic behaviour of the closed-loop system is given by the matrix

$$A(\alpha) = A + B\Big(\int_0^\alpha G(\xi)\, d\xi\Big) C \tag{1.3}$$

whose spectrum is a function of the parameter α. With $\alpha = 0$ we have the open-loop system, whose spectrum is denoted by $\Lambda(0)$. As α varies, the class of linear systems is generated. Every element of the class (the linear system which has the spectrum $\Lambda(\alpha) = \{p_1(\alpha), p_2(\alpha), \ldots, p_n(\alpha)\}$) is defined by a specific value of the parameter α.

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Given the spectrum $\sigma = \{\eta_1, \eta_2, \ldots, \eta_n\}$, into which we wish to transform the spectrum $\Lambda(0)$. We define the Euclidean measure of the mismatch between spectra $\Lambda(\alpha)$ and σ as

$$V(\alpha) = \sum_{i=1}^{n} (p_i(\alpha) - \eta_i)(p_i(\alpha) - \eta_i)^* \tag{1.4}$$

(the asterisk denotes the complex conjugate). We wish to construct the matrix function $G(\xi)$ in such a way that, for some $\bar{\alpha}$ we have $V(\bar{\alpha}) = \min_\alpha V(\alpha)$. In particular, we may have $V(\bar{\alpha}) = 0$. In this case, we shall say that the spectrum σ can be reached from the spectrum $\Lambda(0)$.

2. Equations for the spectrum. Synthesis of the matrix function $G(\xi)$. We consider the matrix $A(\alpha)$ whose spectrum is $\Lambda(\alpha)$. Let $l_k(\alpha)$ and $e_k(\alpha)$ be the left and right eigenvectors of the matrix $A(\alpha)$, corresponding to the eigenvalue $p_k(\alpha)$. Their dynamic behaviour when the parameter α varies is determined by the conditions of the theorem:

Theorem 1. Let the matrix $A(\alpha)$ given by Eq.(1.3) be simple. Then, the variation of its spectrum, and of its system of right and left eigenvectors, is subject to the equations

$$\begin{gathered} dp_k(\alpha)/d\alpha = l_k^T(\alpha)\, U(\alpha)\, e_k(\alpha), \quad p_k(0) = p_k^\circ \\ de_k(\alpha)/d\alpha = F_k(\alpha)\, U(\alpha)\, e_k(\alpha), \quad e_k(0) = e_k^\circ \\ dl_k^T(\alpha)/d\alpha = l_k^T(\alpha)\, U(\alpha)\, F_k(\alpha), \quad l_k(0) = l_k^\circ \\ U(\alpha) = BG(\alpha)C \in R^{n\times n}, \quad F_k(\alpha) = \\ \sum_{i=1, i\neq k}^{n} (p_k(\alpha) - p_i(\alpha))^{-1} Z_i(\alpha) \in E^{n\times n} \\ Z_i(\alpha) = e_i(\alpha)\, l_i^T(\alpha) \in E^{n\times n} \end{gathered} \tag{2.1}$$

where T denotes transposition, p_k°, e_k°, l_k° are the k-th eigenvalue, and k-th right and left eigenvectors of the corresponding matrix of the open-loop object $A = A(0)$, the matrix $Z_i(\alpha)$ is the spectral projector of the corresponding eigenvalue $p_i(\alpha)$, and E^n is the linear n-dimensional space over the complex number field.

Proof. Consider the eigenvalue problem

$$A(\alpha)\, e_k(\alpha) = p_k(\alpha)\, e_k(\alpha) \tag{2.2}$$

On differentiating (2.2) with respect to α and collecting like terms, we get

$$(A(\alpha) - p_k I)\frac{de_k(\alpha)}{d\alpha} = -\left(\frac{dA(\alpha)}{d\alpha} - \frac{dp_k(\alpha)}{d\alpha} I\right) e_k(\alpha) \tag{2.3}$$

where $I \in R^{n\times n}$ is the identity matrix. Noting that $l_k^T(\alpha)(A(\alpha) - p_k(\alpha) I) = 0$, and the normalization $l_k^T(\alpha)\, e_k(\alpha) = 1$, and multiplying (2.3) on the left by $l_k^T(\alpha)$, we obtain the first equation of (2.1).

On substituting this equation into (2.3), and noting that $dA(\alpha)/d\alpha = U(\alpha)$, we transform (2.3) to the form

$$(p_k(\alpha)\, I - A(\alpha))\, de_k(\alpha)/d\alpha = (U(\alpha) - l_k^T(\alpha)\, U(\alpha)\, e_k(\alpha)\, I)\, e_k(\alpha) \tag{2.4}$$

We know /2/ that the following spectral expansions hold for a simple matrix $A(\alpha)$ and its resolvent $R_\alpha(p) = (pI - A(\alpha))^{-1}$:

$$A(\alpha) = \sum_{k=1}^{n} p_k(\alpha)\, Z_k(\alpha), \quad R_\alpha(p) = \sum_{k=1}^{n} (p - p_k(\alpha))^{-1} Z_k(\alpha)$$

We write $R_\alpha(p)$ as

$$\begin{gathered} R_\alpha(p) = (p - p_k(\alpha))^{-1} Z_k(\alpha) + F_k(p, \alpha) \\ F_k(p, \alpha) = \sum_{i=1, i\neq k}^{n} (p - p_i(\alpha))^{-1} Z_i(\alpha) \end{gathered}$$

The function $F_k(p, \alpha)$ is obviously analytic in the neighbourhood of $p_k(\alpha)$. Hence the matrix function $F_k(p, \alpha)(pI - A(\alpha))$ is also analytic in the neighbourhood of $p_k(\alpha)$. For this matrix function, using the spectral resolution of the matrix $A(\alpha)$, and the well-known property of projection matrices $Z_k(\alpha)\, Z_j(\alpha) = \delta_{kj} Z_k(\alpha)$, we obtain the value at the point $p = p_k(\alpha)$

$$F_k(\alpha)(p_k(\alpha)\, I - A(\alpha)) = I - Z_k(\alpha) \tag{2.5}$$

where $F_k(\alpha) = F_k(p_k, \alpha)$. On multiplying (2.4) on the left by $F_k(\alpha)$, and taking account of (2.5), we arrive at the equation

$$(I - Z_k(\alpha))\, de_k(\alpha)/d\alpha = F_k(\alpha)(U(\alpha) - l_k^T(\alpha)\, U(\alpha)\, e_k(\alpha)\, I)\, e_k(\alpha) \tag{2.6}$$

Consider the vector $Z_k(\alpha)\, de_k(\alpha)/d\alpha = e_k(\alpha)\, l_k^T(\alpha)\, de_k(\alpha)/d\alpha$. Since the vector $e_k' = e_k + de_k$

is defined only up to a non-zero multiplicative constant, we define e_k' in such a way that $l_k^T e_k' = l_k^T (e_k + de_k) = 1$. Since $l_k^T e_k = 1$, we have $l_k^T de_k = 0$. Hence $Z_k(\alpha)\, de_k(\alpha)/d\alpha = 0$.

Consider the vector

$$F_k(\alpha)(l_k^T(\alpha) U(\alpha) e_k(\alpha)) e_k(\alpha) =$$
$$l_k^T(\alpha) U(\alpha) e_k(\alpha) \sum_{i=1,\, i\neq k}^{n} (p_k(\alpha) - p_i(\alpha))^{-1} e_i(\alpha) l_i^T e_k(\alpha)$$

Since the right and left eigenvectors with indices $i \neq k$ are orthogonal, we have $l_i^T(\alpha) e_k(\alpha) = 0$. Hence $(l_k^T(\alpha) U(\alpha) e_k(\alpha)) \cdot F_k(\alpha) e_k(\alpha) = 0$.

Using these last two results, we obtain from (2.6) the second equation of system (2.1). The third equation can be obtained in a similar way.

Eq.(2.1) give the trajectories described by the eigenvalues $p_k(\alpha)$ in the complex plane as the parameter α increases from the value $\alpha = 0$. The trajectories start on the spectrum $\Lambda(0)$ corresponding to the open-loop object. For these trajectories, the "control" is the matrix function $G(\alpha)$, which has to be chosen in such a way that the trajectories converge to the desired spectrum of the closed-loop system σ.

In essence, this problem amounts to a generalization of the familiar root hodograph method /3/, which is often used in practice. Here, however, instead of varying one chosen parameter of the closed-loop system, the entire matrix of the feedback is varied. Moreover, the aim of the control is given as a wanted spectrum.

Let us now sythesize the matrix function $G(\alpha)$. For this, we calculate the total derivative of the positive definite function $V(\alpha)$ with respect to α in the light of Eqs. (2.1). After transformations, we obtain

$$dV(\alpha)/d\alpha = s^T(\alpha)\, g(\alpha), \quad V(0) = V_0 \tag{2.7}$$
$$s(\alpha) = h(\alpha) + h^*(\alpha) \in R^{ml},$$
$$h(\alpha) = (C^T \otimes B) \sum_{i=1}^{n} (p_i(\alpha) - \eta_i)^* (e_i(\alpha) \otimes l_i(\alpha))$$
$$g(\alpha) = \operatorname{col} \| (g_{11}(\alpha), \ldots, g_{1l}(\alpha), g_{21}(\alpha), \ldots, g_{2l}(\alpha), \ldots$$
$$\ldots g_{m1}(\alpha), \ldots, g_{ml}(\alpha)) \in R^{ml}$$

where V_0 is the mismatch of the spectra $\Lambda(0)$ and σ, the vector $g(\alpha)$ is composed of the elements $g_{ij}(\alpha)$ of the matrix $G(\alpha)$, and $\otimes$ denotes the direct (Kronecker) product of matrices /2/.

On finding the vector $g(\alpha)$ as

$$g(\alpha) = -\gamma s(\alpha)/\| s(\alpha)\|^2, \quad \| s(\alpha)\|^2 = s^T(\alpha)\, s(\alpha) \tag{2.8}$$

where $\gamma > 0$ is a constant, we have

$$dV(\alpha)/d\alpha = -\gamma < 0 \tag{2.9}$$

In short, when condition (2.8) holds, $V(\alpha)$ is the Lyapunov function, specified on the spectrum of the linear system. By specifying the matrix function $G(\alpha)$ in the form (2.8), we can ensure that the Euclidean measure of the mismatch between the running spectrum $\Lambda(\alpha)$ and the wanted spectrum σ of the closed-loop system decreases monotonically as the parameter α increases from the value $\alpha = 0$.

Notice that the approximation of the spectra $\Lambda(\alpha)$ and σ is not of an asmyptotic type. For, the solution of Eq.(2.9) is the function $V(\alpha) = V(0) - \gamma\alpha$. If $\Lambda(\alpha)$ reaches the desired value σ, we have $V(\bar{\alpha}) = 0$. Hence $\bar{\alpha} = V(0)/\gamma$. Thus, if the spectrum σ is reached from the spectrum $\Lambda(0)$, this occurs after a finite "time" $\bar{\alpha}$.

3. On the spectral control of the method. It follows from (2.7) and (2.8) that the spectrum σ can be reached from $\Lambda(0)$ with our modal control method if the vector $s(\alpha) \neq 0$, $\forall \alpha \in [0, \bar{\alpha})$. If, as the parameter α increases from the value $\alpha = 0$, the vector $s(\alpha)$ becomes zero at some intermediate point $\bar{\alpha}_c \in [0, \bar{\alpha})$, the modal control process breaks off. Then, $V(\bar{\alpha}_c) \neq 0$, so that $\Lambda(\bar{\alpha}_c) \neq \sigma$. Since, as α increases, $V(\alpha)$ decreases monotonically, i.e., $V(\bar{\alpha}_c) < V(\alpha)$, $\forall \alpha \in [0, \bar{\alpha}_c)$, then, in the context of our method, the minimum of the Euclidean measure of the mismatch of the running and wanted spectra is reached at the point $\alpha = \bar{\alpha}_c$.

Consider the conditions under which the vector $s(\alpha)$ becomes zero. Let the wanted spectrum be chosen in such a way that the complex conjugate numbers $p_i(0)$ and $p_j(0) = p_i^*(0)$ are associated with the complex conjugate numbers η_i and $\eta_j = \eta_i^*$, and the real numbers $p_k(0)$ with the real numbers η_k. Noting our assumption that the spectrum $\Lambda(\alpha)$ is simple, this condition is not restrictive, since obviously, when it does not hold, the spectrum $\Lambda(\alpha)$, which contains multiple eigenvalues, will be reached as α increases.

In view of our choice of the spectrum σ, and of the fact that, to the real numbers $p_k(\alpha)$ there correspond real right and left eigenvectors, and to a pair of complex conjugate numbers

$p_i(\alpha)$, $p_j(\alpha) = p_i^*(\alpha)$, a pair of complex conjugate right and left eigenvectors, we see that: the terms in $h(\alpha)$ with real $p_k(\alpha)$ will be real, and the terms with complex conjugate $p_i(\alpha)$, $p_j(\alpha) = p_i^*(\alpha)$ will be complex conjugate. Hence it follows that $h(\alpha)$ is a real vector and $s(\alpha) = 2h(\alpha)$. Instead of $s(\alpha)$, therefore, we can consider the vector $h(\alpha)$, which can be written as

$$h(\alpha) = \Phi(\alpha)\,\Delta p^*(\alpha), \quad \Delta p(\alpha) = p(\alpha) - \eta \in E^n, \quad \Phi(\alpha) \in E^{ml\times n} \tag{3.1}$$

$$p(\alpha) = \mathrm{col}\,(p_1(\alpha),\ p_2(\alpha), \ldots, p_n(\alpha)), \quad \eta = \mathrm{col}\,(\eta_1, \eta_2, \ldots, \eta_n)$$

Here, c_i is the vector consisting of the elements of the i-th row of matrix C, and b_j is the vector consisting of the elements of the j-th column of matrix B.

$$\Phi(\alpha) = \left\| \begin{array}{cccc} c_1^TZ_1b_1 & c_1^TZ_2b_1 & \ldots & c_1^TZ_nb_1 \\ c_2^TZ_1b_1 & c_2^TZ_2b_1 & \ldots & c_2^TZ_nb_1 \\ \vdots & \vdots & & \vdots \\ c_l^TZ_1b_1 & c_l^TZ_2b_1 & \ldots & c_l^TZ_nb_1 \\ \hline c_1^TZ_1b_2 & c_1^TZ_2b_2 & \ldots & c_1^TZ_nb_2 \\ c_2^TZ_1b_2 & c_2^TZ_2b_2 & \ldots & c_2^TZ_nb_2 \\ \vdots & \vdots & & \vdots \\ c_l^TZ_1b_2 & c_l^TZ_2b_2 & \ldots & c_l^TZ_nb_2 \\ \hline \vdots & \vdots & & \vdots \\ \hline c_1^TZ_1b_m & c_1^TZ_2b_m & \ldots & c_1^TZ_nb_m \\ c_2^TZ_1b_m & c_2^TZ_2b_m & \ldots & c_2^TZ_nb_m \\ \vdots & \vdots & & \vdots \\ c_l^TZ_1b_m & c_l^TZ_2b_m & \ldots & c_l^TZ_nb_m \end{array} \right\|$$

Let rank $\Phi(\alpha) = n$, $\forall\alpha \in [0, \bar{\alpha})$. Then, the annihilated subspace $N(\Phi(\alpha))$ of the matrix $\Phi(\alpha)$ consists only of the zero element $\Delta p(\alpha) = 0$ /2/. Hence the vector $h(\alpha) \neq 0$, $\forall\alpha \in [0, \bar{\alpha})$ and the modal control process only ends when the wanted spectrum σ is reached. We have thus proved the following theorem for a rank criterion for spectral control of a closed-loop system.

Theorem 2. Let the object (1.1) with the spectrum $\Lambda(0)$ be closed by the feedback (1.2), and let the wanted spectrum σ of the closed-loop system be given. Then, if

$$\mathrm{rank}\,\Phi(\alpha) = n, \quad \forall\alpha \in [0, \bar{\alpha}) \tag{3.2}$$

the spectrum σ can be reached from the spectrum $\Lambda(0)$.

If rank $\Phi(\alpha) < n$, then non-zero $\Delta p(\alpha) \in N(\Phi(\alpha))$ are solutions of the equation $\Phi(\alpha)\,\Delta p^*(\alpha) = 0$. Since $\Delta p(\alpha) = p(\alpha) - \eta$, this means that, for the running spectrum $\Lambda(\alpha)$ in the space E^n, there exists a manifold $H(\alpha)$ such that, if $\eta \in H(\alpha)$, then $h(\alpha) = 0$. Hence it follows that, if the vector η belongs to the manifold $H(\alpha)$ for some $\alpha \in [0, \bar{\alpha})$, then the corresponding spectrum σ cannot be reached from the spectrum $\Lambda(0)$ by the present method.

Condition (3.2) embraces the conventional requirement that the closed-loop system be completely controllable and observable $\forall\alpha \in [0, \bar{\alpha})$. For, if this requirement is infringed, there must be a right $e_i(\alpha)$ or left $l_j(\alpha)$ eigenvector of the matrix $A(\alpha)$ such that: either $Ce_i(\alpha) = 0$, or else $l_j^T(\alpha)B = 0$ /4/. In this case, the corresponding column of the matrix $\Phi(\alpha)$ becomes zero, and rank $\Phi(\alpha) < n$. Condition (3.2) also includes a bound on the number m of inputs and l of outputs of the object (1.1):

$$ml \geqslant n \tag{3.3}$$

where n is the dimensionality of the object. If we have the reverse, then automatically rank $\Phi(\alpha) \leqslant ml < n$. It must be said that condition (3.2) is different from the well-known condition /5/: $n \leqslant m + l - 1$. Given n, condition (3.2) allows fewer inputs and outputs, which is important from the practical point of view.

Furthermore, condition (3.2) is not reducible to the satisfaction of these two requirements. For a completely controlled and observed system with $ml \geqslant n$ there is a set of distributions of the eigenvectors which cause all the minors of the matrix $\Phi(\alpha)$ of rank n to vanish.

The following example shows that this set of distributions is not empty. Let $n = 4$, $l = 2$, $m = 3$, and let the eigenvectors be such that $c_1^Te_1(\alpha) = 0$, $c_1^Te_2(\alpha) = 0$, $c_2^Te_3(\alpha) = 0$, $c_2^Te_4(\alpha) = 0$, $l_3^T(\alpha)b_1 = 0$, $l_4^T(\alpha)b_1 = 0$, $l_3^T(\alpha)b_2 = 0$, $l_4^T(\alpha)b_2 = 0$, $l_1^T(\alpha)b_3 = 0$, $l_2^T(\alpha)b_3 = 0$. On writing the matrix $\Phi(\alpha)$ for this case, we see that rank $\Phi(\alpha) = 3$.

4. Example.

To illustrate our method, we will consider the problem of achieving the maximum degree of stability in the system "undamped harmonic oscillator plus aperiodic element" described by the equations

$$\begin{Vmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Vmatrix} = \begin{Vmatrix} 1 & 1/2 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & 0 \end{Vmatrix} \begin{Vmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{Vmatrix} + \begin{Vmatrix} 1 \\ 0 \\ 1 \end{Vmatrix} u(t) \quad (4.1)$$

where $x_1(t)$ is the output of the aperiodic section (the observed variable), and $x_2(t)$, $x_3(t)$ are respectively the coordinate and rate of change of the harmonic oscillator (the unobserved variables). For the object (4.1) we take the control law

$$u(t) = \left(\int_0^\alpha g_1(\xi)\,d\xi\right) x_1(t) + \left(\int_0^\alpha g_2(\xi)\,d\xi\right) x_3(t), \quad \dot{x}_4(t) = -20x_4(t) + x_1(t) \quad (4.2)$$

Obviously, the problem of control by the dynamic controller (4.2) of a simple extension of the state space of object (4.1) amounts to problem (1.1), (1.2).

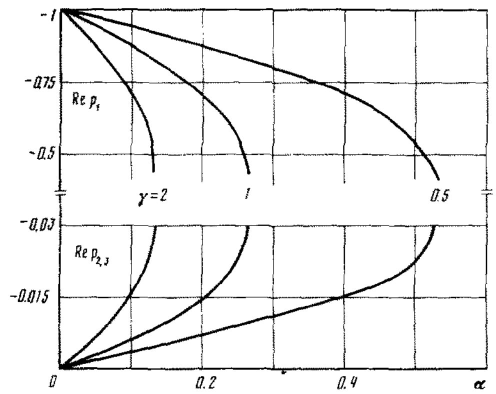

Fig.1

With $\alpha = 0$ in the open-loop position we have the spectrum value: $\Lambda(0) = \{p_1(0) = -1,\ p_2(0) = j2,\ p_3(0) = -j2,\ p_4(0) = -20\}$. The aim of the control is to shift the poles $p_2(\alpha)$ and $p_3(\alpha)$ leftwards parallel to the real axis of the complex plane through as large a distance as possible. As the wanted spectrum we take $\sigma = \{\eta_1 = -1,\ \eta_2 = -5 + j2,\ \eta_3 = -5 - j2,\ \eta_4 = -20\}$.

Simulation of the continuous modal control process with Lyapunov frequency function (1.4) with $n = 4$ and algorithm (2.8) for synthesizing the vector function $g(\alpha) = \mathrm{col}\,(g_1(\alpha), g_2(\alpha))$, showed that the poles $p_2(\alpha)$ and $p_3(\alpha)$ are displaced leftwards parallel to the real axis of the complex plane, while the real pole $p_1(\alpha)$ is shifted rightwards, and the real pole $p_4(\alpha)$ remains virtually fixed.

The pole displacement is shown in Fig.1, where the curves $\mathrm{Re}\,p_1$, and $\mathrm{Re}\,p_{2,3}$ are plotted against α for different values of the coefficient γ. It can be seen that the running spectrum $\Lambda(\alpha)$ does not reach the wanted value σ. At an intermediate point $\bar{\alpha}_c \in [0, V(0)/\gamma)$, whose value depends on γ, the vector $s(\alpha)$ becomes zero and the control process breaks off. The spectrum $\Lambda(0)$ then transforms into $\Lambda(\bar{\alpha}_c) = \{p_1(\bar{\alpha}_c) = -0.45,\ p_2(\bar{\alpha}_c) = -0.03 + j2,\ p_3(\bar{\alpha}_c) = -0.03 - j2,\ p_4(\bar{\alpha}_c) = -20\}$, which is independent of the coefficient γ.

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